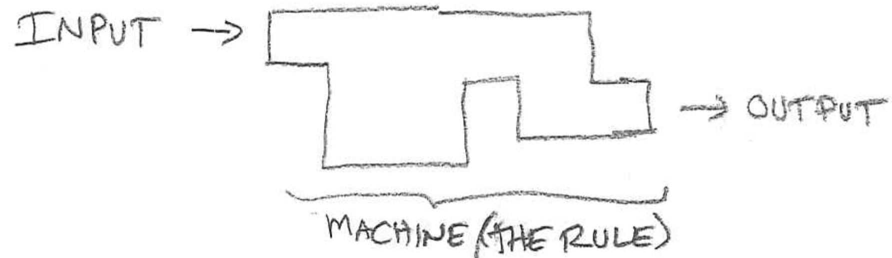
Closing Tue: Sup. 1-3, 4
Closing Thur: Sup. 5

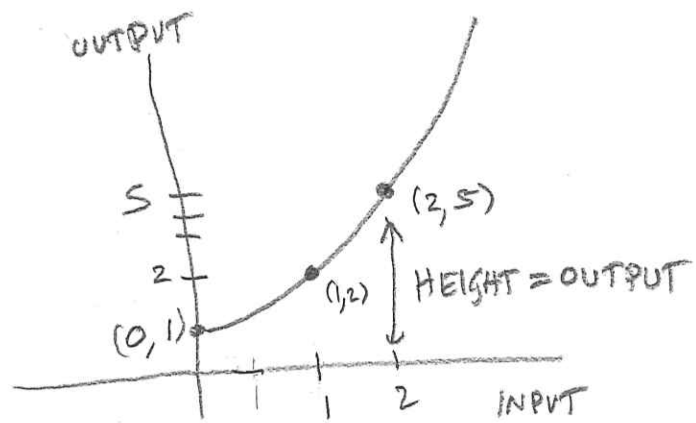
***Today*: Sup. 5 Functional Notation**
Get out graph/table that go with Sup. 5 (pages 4-6 of lecture pack)

Def'n: A **function** is a rule that produces a single output for every allowable input.



IT CAN BE GIVEN AS A TABLE, OR GRAPH, OR FORMULA.

INPUT	OUTPUT
0	1
1	2
2	5
3	10
4	17



$1 + (\text{INPUT})^2 = \text{OUTPUT}$

WE USUALLY GIVE THE INPUT/OUTPUT SYMBOLS

$x = \text{INPUT}, \quad y = \text{OUTPUT}$

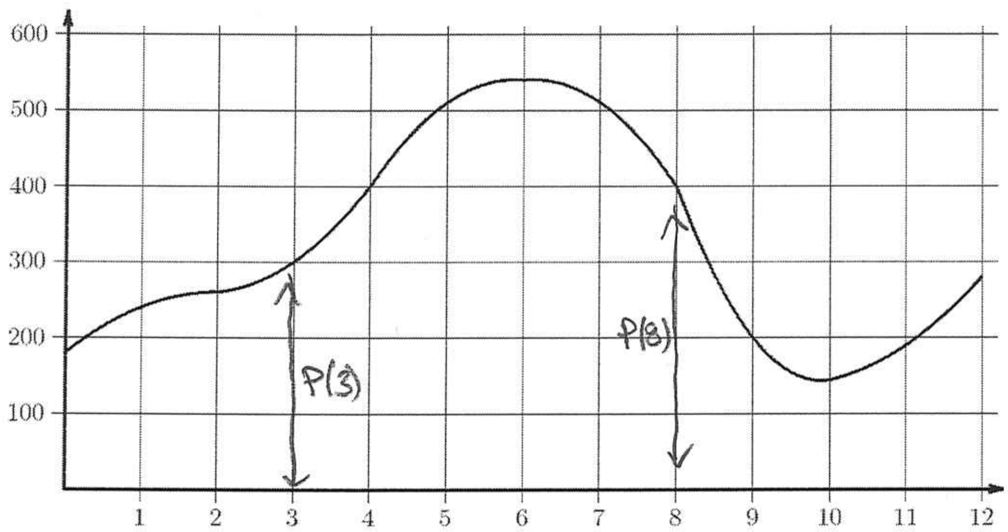
AND WE OFTEN GIVE THE FUNCTION A NAME

Silly example: FRED and we write

$y = \underbrace{\text{FRED}(x)}$

MEANS VALUE YOU GET FROM x AFTER PLUGGING INTO THE FRED MACHINE.

Temp vs time for a chem. reaction



For the rest of today, we will practice *translating* between

1. Functional notation
2. Graphs (values, heights, slopes)
3. English (time, temp, rates)

Let t = time (in minutes)

P = temperature (in ^{o}C)

P(t) = "temperature at time t"

$P(0) \approx 180$

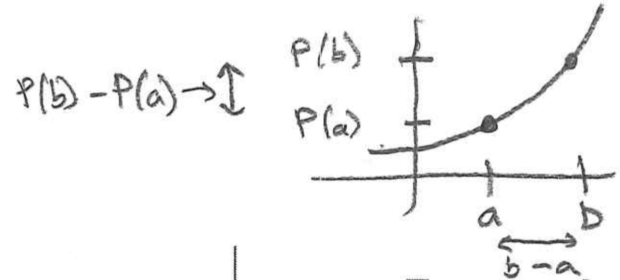
$P(3) = 300$

$P(8) = 400$

$P(8) - P(3) = 100$, WHAT IS THIS IN THE GRAPH?

CHANGE IN HEIGHT FROM 3 TO 8.

Consider rows 1-6 of the lecture pack, fill in the translations.



	English	Graph	Functional Notation
1	At time $t = 4$, the temperature is 400°.	At $t = 4$, the height of graph is 400.	$P(4) = 400$
2	CHANGE IN TEMP From $t=a$ to $t=b$	CHANGE IN HEIGHT From $t=a$ to $t=b$	$P(b) - P(a)$
3	the incremental rate of change in temperature from time a to time b	SLOPE OF SECANT LINE THRU $t=a$ AND $t=b$ ON GRAPH	$\frac{P(b) - P(a)}{b - a}$
4	temp. at $t=0$ minutes	the "y"-intercept of the temperature graph	$P(0)$
5		SLOPE OF DIAGONAL LINE THRU $t=b$ ON GRAPH	$\frac{P(b)}{b}$
6	OVERALL RATE OF CHANGE OF TEMP ~~At~~ b MINUTES	SLOPE OF SECANT LINE THRU $t=0$ AND $t=b$ ON GRAPH	$\frac{P(b)-P(0)}{b-0}$

NICE & COMPACT!

ALL VERY IMPORTANT!

Very Important Notes:

If f(t) = "height at t", then

$f(b) - f(a)$ = "change in height from *t=a* to *t=b*"

$\frac{f(b)-f(a)}{b-a}$ = "slope of secant line thru *t=a* and t=b" = "inc. ave. rate"

$\frac{f(b)-f(0)}{b-0}$ = "slope of secant line thru *t=0* and *t=b*" = "overall ave. rate"

$\frac{f(b)}{b} = \frac{f(b)-0}{b-0}$ = "slope of diagonal line thru t=b"

Note:

1. If f(0) = 0, then the overall ave. rate is the same as the slope of the diagonal line.

2. a = start of the interval

   b = end of the interval

Try rows 8-14 on your own later

Let's talk about intervals.

Intervals Fact 1:

An important translation

"*h* minutes after *t*": $t + h$

"*h* minutes before *t*": $t - h$

Now try Row 13:

Translate this to functional notation

"Find two times, two minutes apart where the temperature is the same."

Let t = "first time"

Then 2-minutes later is

$t + 2$

WE WANT TO FIND t AND $t+2$ SO THAT

$$P(t) = P(t+2)$$

Flip to the next page of translations.

Intervals Fact 2: Find the start, $t=a$, and the end, $t=b$.

Examples:

"2-minute interval starting at *t*"

start = t   end = t + 2

"*h*-minute interval starting at 3"

start = 3   end = 3 + h

"5-minute interval ending at *b*"

start = *b* – 5   end = *b*

Question:

Translate rows 18, 19, and 20

Also solve row 18.

$\boxed{18}$ $P(3+r) - P(3) = 100$. FIND r.

START = 3, END = 3 + r

WANT "CHANGE IN HEIGHT FROM 3 TO 3 + r TO BE 100"

$P(3) = 300$ SO WE WANT 100 more

$P(3+r) = 400$

THAT HAPPENS WHEN $3+r=4$ or $3+r=8$

$r=1$ or $r=5$

$\boxed{19}$ $\text{SLOPE} = \dfrac{P(t+3) - P(t)}{(t+3) - (t)}$

START = t

END = t + 3

$\boxed{20}$ START = 3   END = 3 + h

$\text{RATE} = \dfrac{P(3+h) - P(3)}{(3+h) - 3}$

Sup. 5 / Problem 4:

The graph of y = f(x) is given.

(a) Compute $\frac{f(8)}{8}$

"slope of diagonal line at 8"

TWO PTS: (0,0) (8,3)

ANSWER $= \frac{3-0}{8-0} = \frac{3}{8} = 0.375$

(b) Compute $\frac{f(19)-f(10)}{9}$

"slope of secant line through 10 & 19"

TWO PTS: (10, 2), (14, 2.5)

ANSWER $= \frac{2.5-2}{14-10} = \frac{0.5}{4} = 0.125$

(c) Find x such that $\frac{f(x)-f(0)}{x} = 0.3$

"SLOPE OF SECANT FROM 0 to x EQUALS 0.3"

↑ GIVEN SLOPE

DRAW A REFERENCE LINE!

(0,0), (1,0.3), ... (10, 3), ..

SLIDE RULER PARALLEL UNTIL TOUCHING GRAPH AT x=0.

ANSWER: $\boxed{x \approx 7.5}$

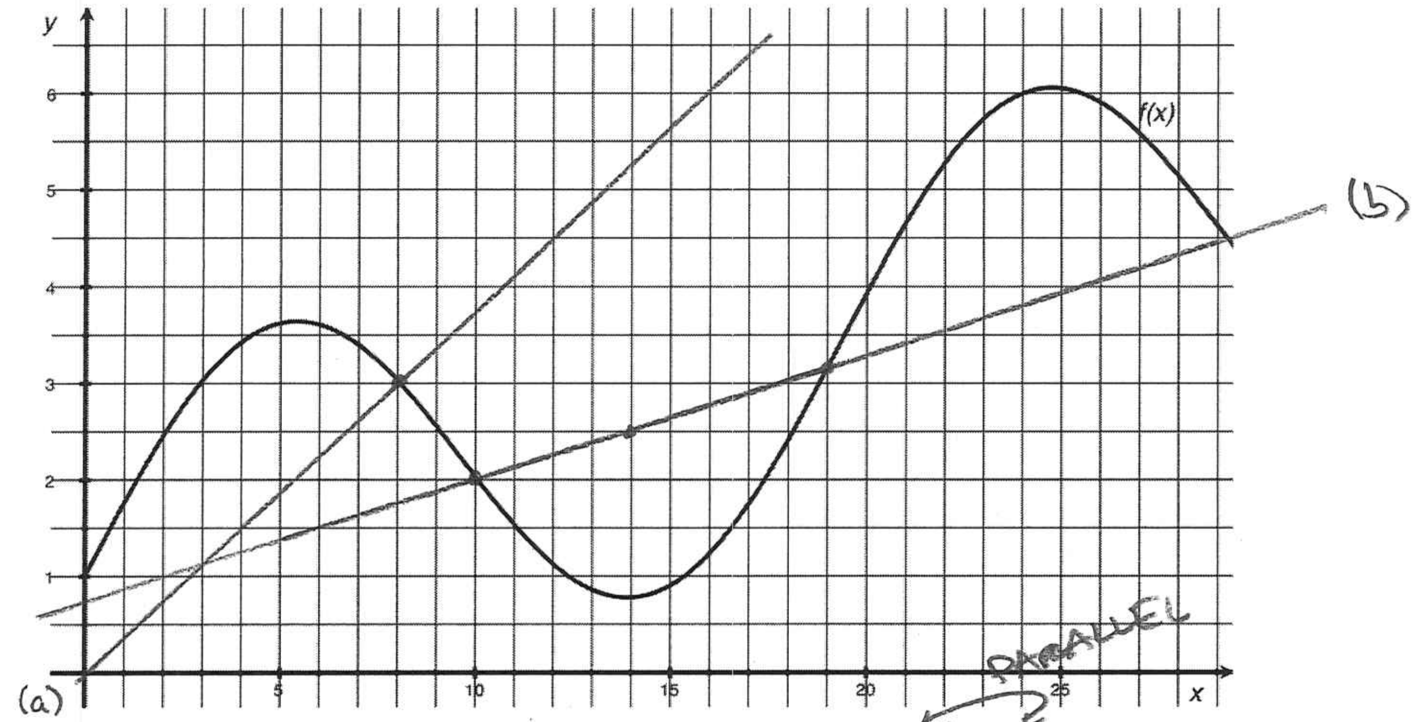
(d) Find x such that $f(x) - f(15) = 2$

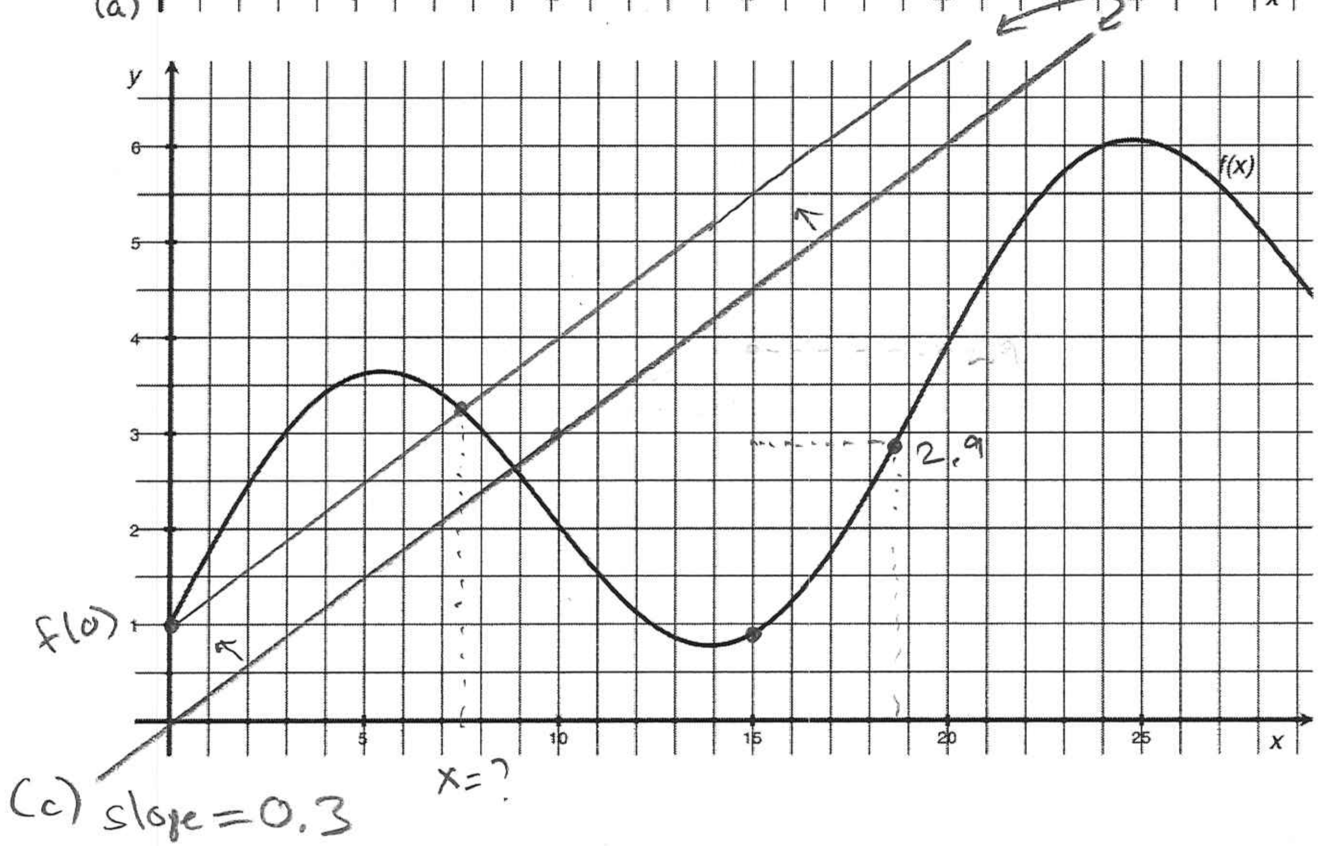
"CHANGE IN HEIGHT FROM 15 TO x EQUALS 2"

START AT 15 AND GO UP 2 IN HEIGHT!

$f(15) \approx 0.9$ SO WE WANT $f(x) \approx 2.9$

ANSWER $x \approx 18.7$


y
6
5
4
3
2
1
5
10
15
20
25
x
f(x)
(a)
(b)
PARALLEL


y
6
5
4
3
2
1
5
10
15
20
25
x
f(x)
f(0)
2.9
x=?
(c) slope=0.3

(h) As x takes on every value from $x = 2$ to $x = 8$, which best describes the values of $\frac{f(x+0.1)-f(x)}{0.1}$?

"SLOPE OF SECANT FROM x TO $x+0.1$"

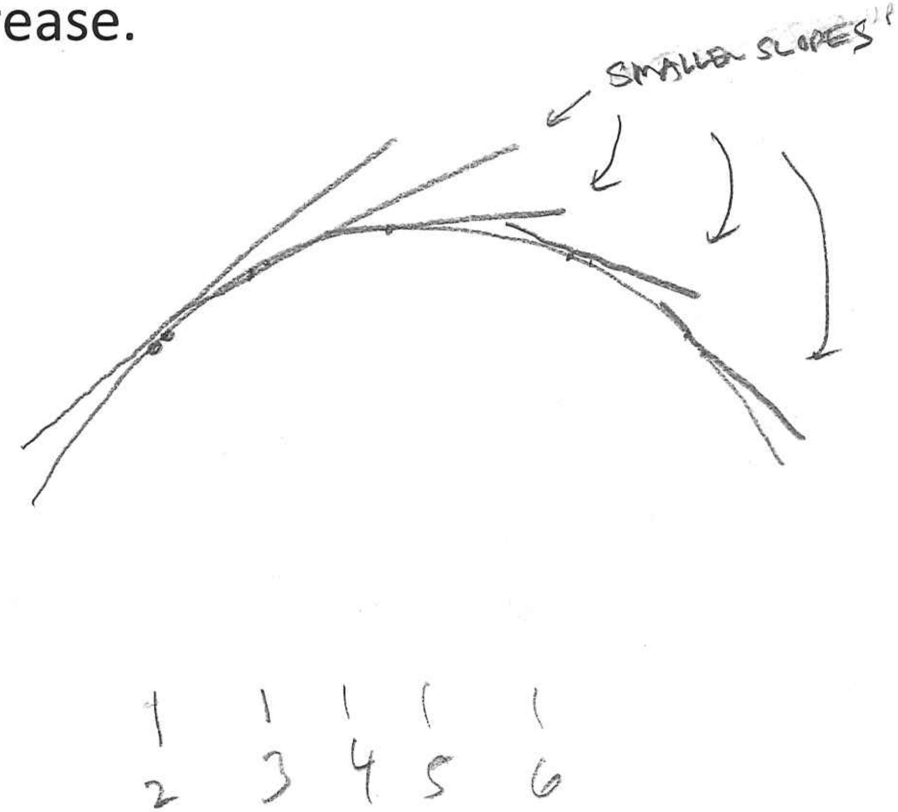
LOOKS LIKE A TANGENT!

i) They increase.

ii) They increase, then decrease.

iii) They decrease.

iv) They decrease, then increase.



	English	Graph	Functional Notation
1	At time $t = 4$, the temperature is 400°.	At $t = 4$, the height of graph is 400.	$P(4) = 400$
2	the change in temp. from time t=a to t=b minutes	the change in the height of the graph from t=a to t=b	$P(b) - P(a)$
3	the incremental rate of change in temperature from time a to time b	the slope of the secant line through the temp. graph at t=a and t=b	$\frac{P(b)-P(a)}{b-a}$
4	the temperature at t=0 minutes	the "y"-intercept of the temperature graph	$P(0)$
5		the slope of the diagonal line through the temp graph at t=b	$\frac{P(b)}{b}$
6	the overall rate of change of temperature after b minutes	the slope of the secant line through the temp. graph at t=0 and t=b	$\frac{P(b)-P(0)}{b-0}$
7	The temp after 10 min is higher than the temp after 9 min.	The graph of temp is higher at t=10 than at t=9.	$P(10) > P(9)$ * * false
8	Between 4 and 6 minutes, the temperature rises by 140°.	Between t=4 and t=6, the temp graph rises 140°.	$P(6) - P(4) = 140$
9	During the first four minutes, the temperature rises on average 57° per minute.	The slope of the secant from $t = 0$ to $t = 4$ is 57.	$\frac{P(4)-P(0)}{4} = 57$
10	When is the temperature 350°?	For what value of t is the height of the temp graph 350?	For what value of t is $P(t)=350$?
11	There are three times when the temp is 200°.	The graph has height 200 for three different values of t.	There are three values of t such that $P(t) = 200$.
12	Find a time at which the temp is more than 100° higher than the temp at t=2 min.	Find a time at which the height of the temp graph is ~~at least~~ more than 100° higher than the height at t=2.	Find t so that $P(t) - P(2) > 100$.
13	Find two times, 2 minutes apart, when the temperature is the same.	Find two times, two min. apart, at which the height of the temp graph is the same.	Find two times, t and t+2, such that $P(t)=P(t+2)$.
14	incremental rate of change in temp from t=2 to t=h minutes	slope of the secant line from 2 to h	$\frac{P(h)-P(2)}{h-2}$.

Super important

	English	Graph	Functional Notation
15	How many minutes after $t = 4$ does the temperature become 250°?	For what value of h is the height of the graph at 4+h equal to 250?	For what value of h is $P(4+h) = 250$?
16	Over which three-minute interval is the change in temp greatest?	For which 3-unit horizontal change is the vertical change in the temp graph greatest?	If $\Delta t = 3$, for what t is ΔP highest?
17	the change in temp over the 2-minute interval beginning at time t	the change in height of the graph between t and $t+2$	$P(t+2) - P(t)$
18	Find all values of r such that the temp at time 3+r is 100° higher than the temp at t=3.	Find the values of r such that the graph at t=3+r is 100° higher than the graph at t=3.	Solve $P(3+r) - P(3) = 100$ for r
19	average rate of change of temp over the 3-minute interval starting at time t	slope of secant line from t to $t+3$.	$\dfrac{P(t+3) - P(t)}{3}$
20	the average rate of change of temperature for h minutes beginning at $t = 3$	slope of secant line from 3 to 3+h	$\dfrac{P(3+h) - P(3)}{h}$
21	average rate of change of temperature from t=p to t=q	slope of the secant from $t = p$ to $t = q$.	$\dfrac{P(q) - P(p)}{q-p}$
22	Do the temperatures at t=2 and t=3 add up to the temp at t=5?	Do the heights at t=2 and t=3 add up to the height at t=5?	Is $P(2) + P(3) = P(5)$?
23	When is the temp twice as high as the temp at t=10 min?	For which t is graph twice as high as it is when $t = 10$?	For which t is $P(t) = 2 \cdot P(10)$?
24	Find a span over which temp rises by 50°/min on average.	Find two times over which the slope of the secant line through the temp graph is 50.	Find two times a and b such that $\dfrac{P(b)-P(a)}{b-a} = 50$.
25	The temp changes faster on average from t=2 to t=5 than it does from t=2 to t=8.	The ~~slope of the~~ secant line through temp from t=2 to t=5 is steeper than the secant from t=2 to t=8.	$\frac{P(5)-P(2)}{5-2} > \frac{P(8)-P(2)}{8-2}$